



**Purdue University**  
**Center for Education and Research in**  
**Information Assurance and Security**



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# SECURE OUTSOURCING

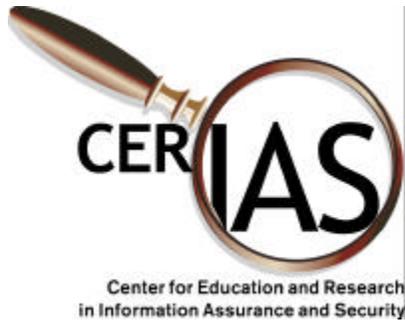
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# DEFINITIONS

$C$  has data and a task (a problem).

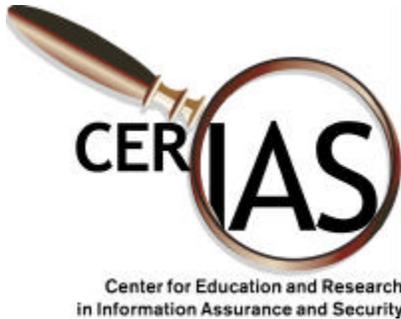
$A$  has hardware and appropriate software (a server).

**Outsourcing:**  $C$  sends problem to  $A$ .

$A$  computes a solution and returns answer to  $C$ .

**Secure Outsourcing:**  $A$  never knows the problem or its solution.

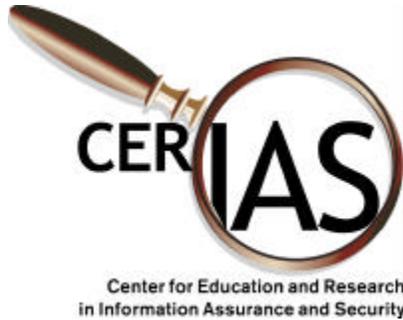
**Mutually Secure Outsourcing:**  $C_1$  and  $C_2$  send parts of the problem to  $A$  who returns the solution to both.  $C_1$  and  $C_2$  do not learn each others problem data,  $A$  still knows nothing.



# SIMPLE OUTSOURCING EXAMPLE

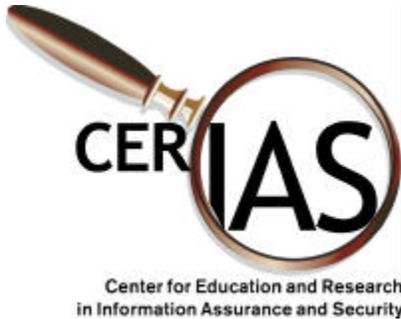
Problem: Compute  $\int_1^a f(x)dx$ .

- Disguise problem by choosing a random function  $s(x)$  of smooth piecewise cubics.
  - Send A the problem: Compute  $\int_1^9 [f(x) + s(x)]dx$  and get  $I$  back.
3. Find answer  $I - \int_1^9 s(x)dx$ . It is trivial to integrate  $s(x)$ .



# SECURE OUTSOURCING REQUIRES

- A cheap way to disguise the problem.
- A cheap way to unveil the answer.
- A disguise that is completely secure.



## A REAL EXAMPLE: Matrix Multiplication $M_1M_2$

**Step 1:** Create 3 random permutations of  $1 \rightarrow N$  and 3 sets of random, non-zero numbers.

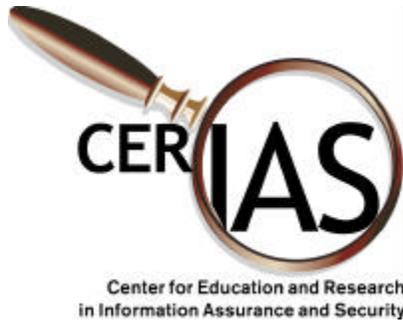
**Step 2:** Put the numbers into the non-zero places of the permutation matrices  $P_1$ ,  $P_2$ , and  $P_3$ .

**Step 3:** Compute  $X = P_1M_1P_2^{-1}$ ,  $Y = P_2M_2P_3^{-1}$ .

**Step 4:** Outsource problem  $Z = XY$ .

**Step 5:** Compute  $P_1^{-1} ZP_3$  which is  $M_1M_2$ .

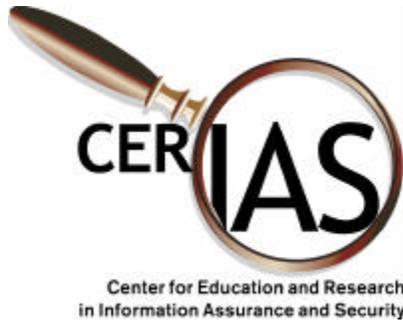
Work of Steps 1, 2, 3 and 5 is order  $N^2$ . Work of Step 4 is order  $N^3$ .  $X$ ,  $Y$  and  $Z$  reveal nothing about  $M_1$  and  $M_2$ .



## Disguise vs. Encryption

Disguise makes a functional or mathematical transformation of the entire object. It preserves distance and is really a “floating point” methodology. Encryption processes the symbols of an object and destroys distance.

Secure Outsourcing is a form of Secure Multi-party Computing which uses disguise instead of the usual encryption.

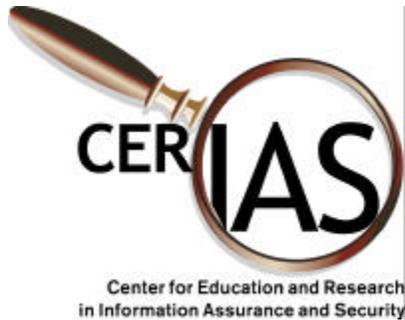


## KEY QUESTION ONE

How general is outsourcing disguise? One can do:

- **Linear algebra**: Multiplication, Inversion, Solve  $Ax = b$ , Convolution, etc.
- **Sorting**.
- **Calculus**: integration, differential equations, transforms.
- **Matching**: Strings, approximate templates, images.

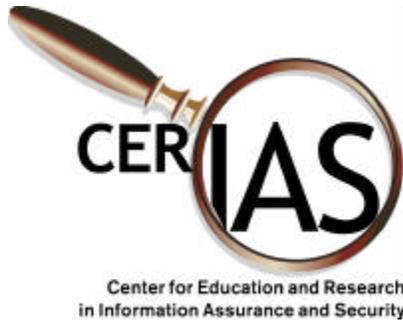
The work to do the disguise and unveil the answer is small compared to the problem solving. The server's work is comparable to the work without disguise.



## KEY QUESTION TWO

How secure is the outsourcing disguise? Our studies suggest the disguises can be made unbreakable to:

- Statistical attacks (try to find the random numbers, functions, mappings,..)
- Approximation theory attacks (try to discover the function behaviors).
- Symbolic code attacks (try to decipher the problem text).



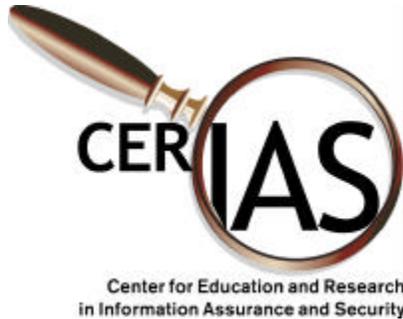
## KEY QUESTION THREE

How broad are the applications of disguise? Is it only applicable to scientific computations?

**Answer 1:** There are some substantial direct applications.

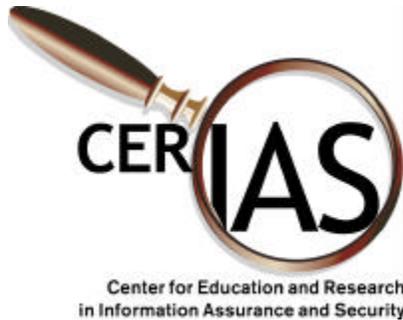
- Schlumberger seismic data analysis service.
- Pattern matching.
  - Was your son caught on the video tapes of the Purdue riots?
  - Is there an oil well being drilled in Benton county?
  - Is a variation of my program being used in MS word?
- Device design.
  - Will your new windmill really be 150% more efficient?

**Answer 2:** Mutually secure disguises have much broader applications.



## MUTUALLY SECURE OUTSOURCING APPLICATIONS

- Joint seismic exploration.
- Joint device design.
  - These are impractical without such a technology.
- Medical diagnosis (records compared to symptoms).
  - Does anyone here have AIDS? Leprosy?
  - What's wrong with my daughter?
- Biometric identification (characteristics compared to records).
  - Do your fingerprints match those on the gun?
  - Are you Mike Atallah?



## KEY QUESTION FOUR

How would this work in practice?

**Sophisticated Users**: A problem solving environment would allow the “programming” of disguises using high level, built-in operators (random functions, key management, symbolic transformations, domain transformations, one time random sequences,...)

**Ordinary Applications**: A fixed disguise program handles everything for a particular application. The only input is a key and the information. The program may be embedded in a device.