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Compressed Sensing for Enhanced Space Security: Resolving Details of Space Objects

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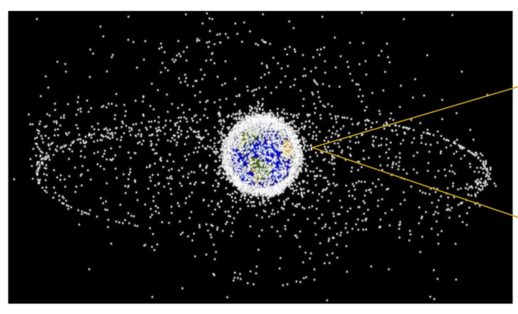
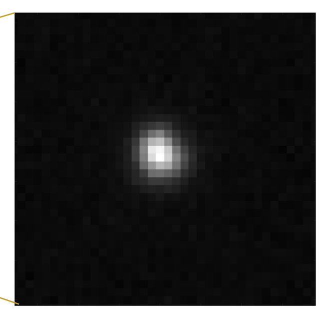
1. Background

Problem: The need for space surveillance has been increasing due to the large number of artificial objects in low Earth orbit (*LEO*). However, most objects are too small or distant to be resolved by ground-based optical telescopes.

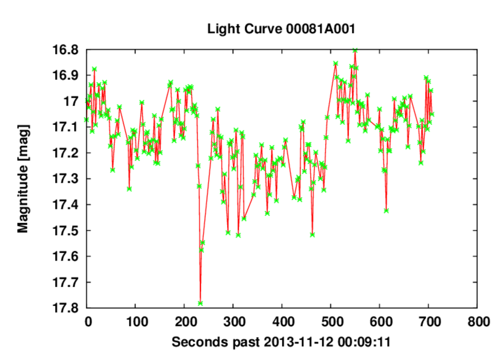
Proposed solution: Using a time history of brightness measurement (*light curve*) of a space object as a clue to estimate the object's shape.



Artificial objects around the Earth



Nonresolved image



Light curve

2. Methods

2.1 Adaptation to compressed sensing

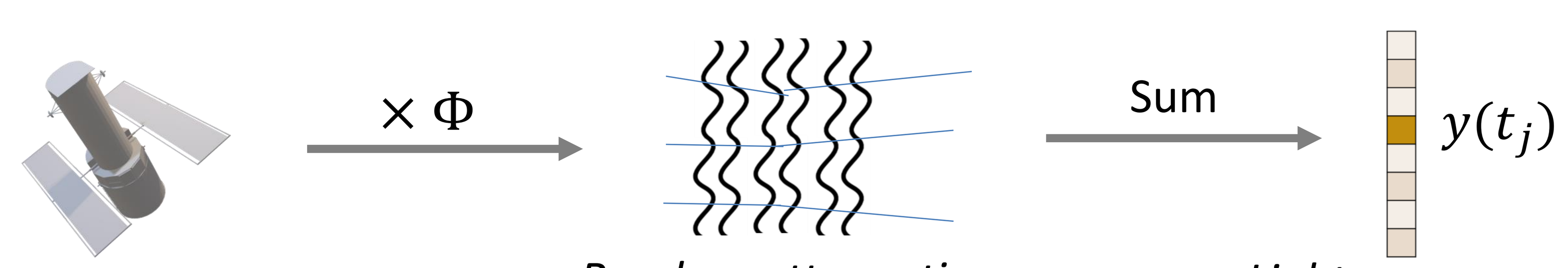
Image recovery in compressed sensing

Compressed sensing is an image processing framework that recovers images from limited samples that are even fewer than Nyquist rate. *Sensing matrix* Φ is used as an encryption key to compress and recover an image.

$$\hat{\boldsymbol{\gamma}} = \arg\min \|\boldsymbol{\gamma}\|_1 \ \text{s.t.}\ \|\boldsymbol{y} - \boldsymbol{\Phi\Psi\gamma}\|_2 \le \varepsilon,$$
$$\hat{\boldsymbol{x}} = \boldsymbol{\Psi}\hat{\boldsymbol{\gamma}} \ \text{ where } \boldsymbol{\Psi}\text{: Dictionary}$$

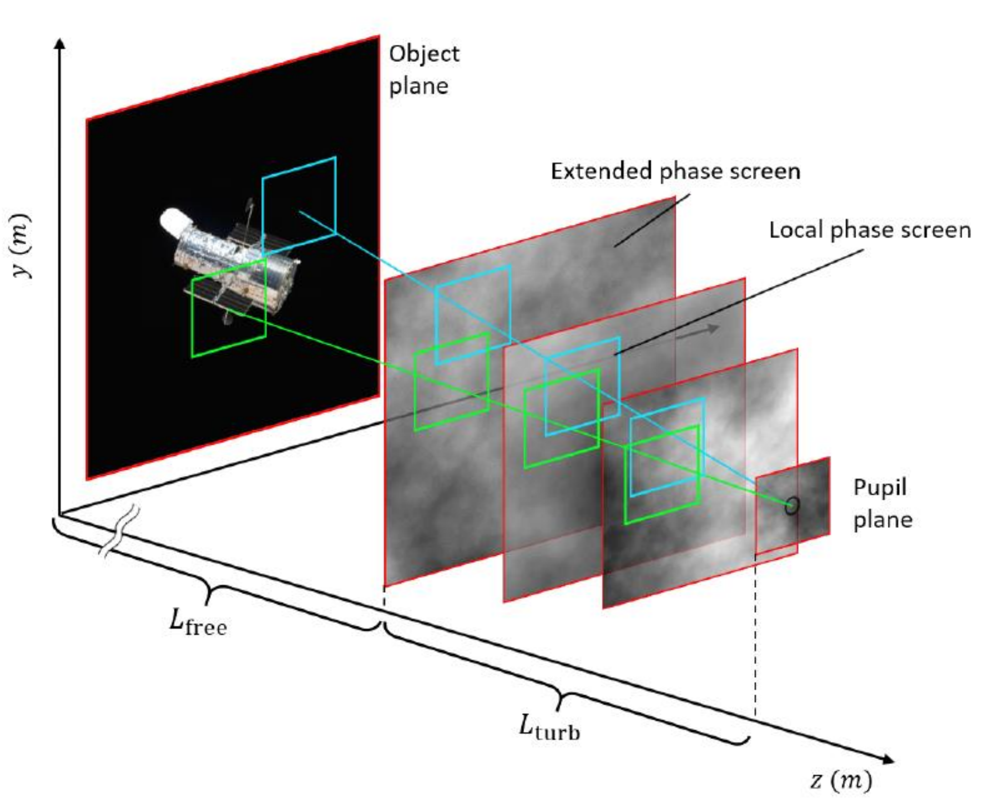
Analogy to light curve measurement

This study has shown that a light curve of a LEO object can be modeled as the compressed image. The sensing matrix, Φ, represents the superposition of *point spread function (PSF),* which we call a *PSF map.*



$$\hat{\boldsymbol{\gamma}}_I = \arg\min \|\boldsymbol{\gamma}_I\|_1 \ \text{s.t.}\ \|\boldsymbol{D}_I^*(\boldsymbol{y} - \boldsymbol{D}_I\boldsymbol{\gamma}_I)\|_\infty \le \varepsilon,$$
$$\hat{\boldsymbol{x}}_I = \boldsymbol{\Psi}_I\hat{\boldsymbol{\gamma}}_I \ \text{ where } \boldsymbol{D}_I = \boldsymbol{\Phi}_I\boldsymbol{\Psi}_I$$

2.2 Atmospheric modeling by wave propagation



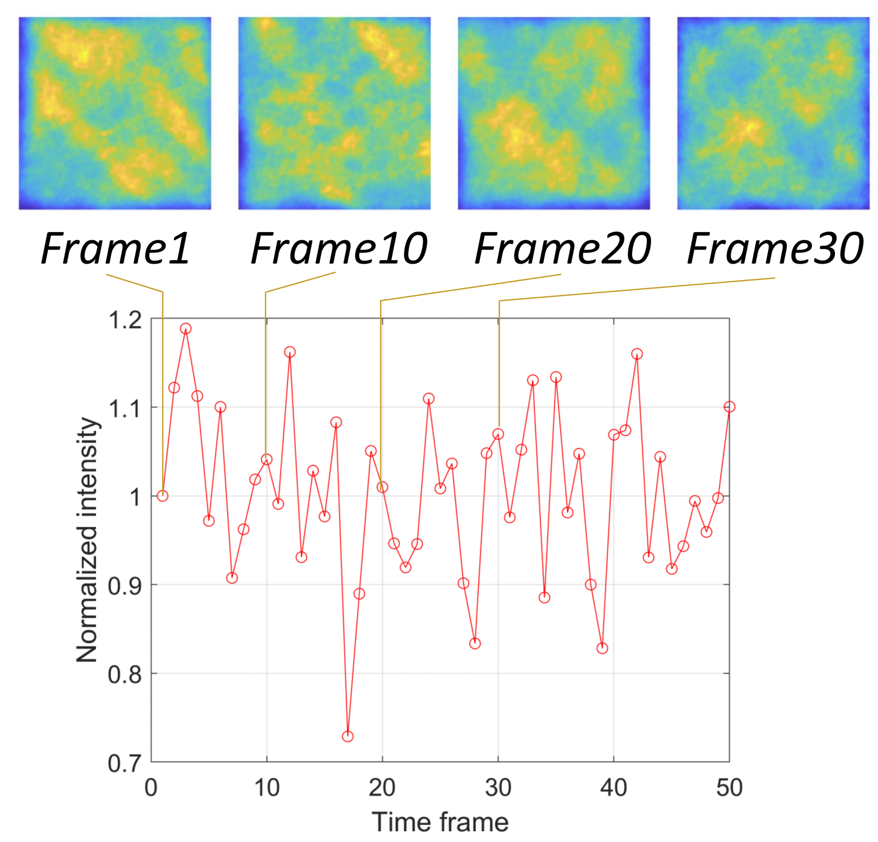
Schematic of optical wave propagation method

Spherical wave from a sample point
↓
Propagation through free space
↓
Propagation through turbulence layers
↓
Point Spread Function (PSF) and light curve

3. Simulation and Results

3.1 Numerical wave propagation

Time history of PSF maps



Light curve with fast wind speed

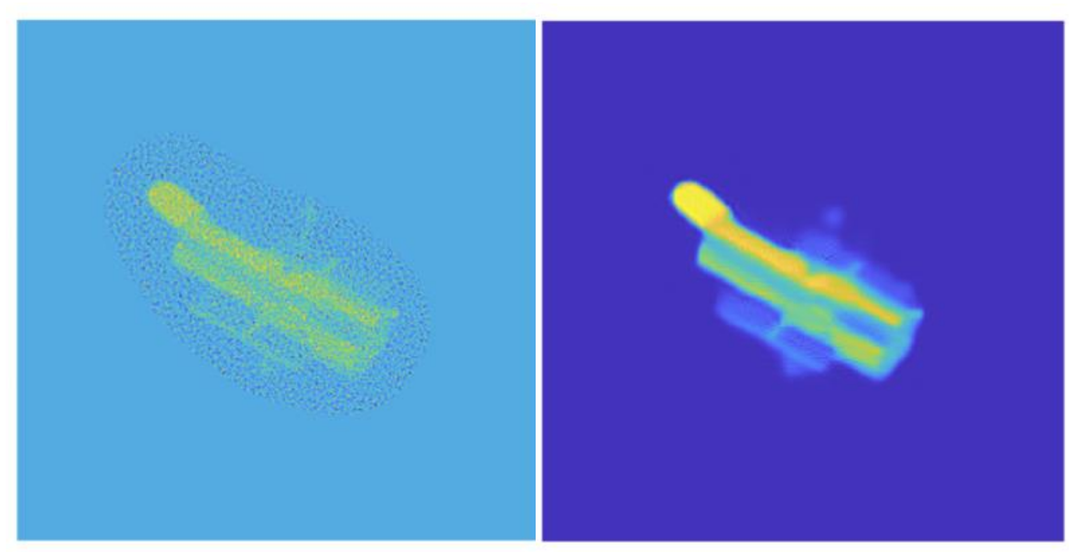
Simulation parameters of the sensor and the wave propagation through turbulence

Parameter	Value
Aperture diameter PSF sensor	$D = 0.61$ m
Focal length PSF sensor	$f_\ell = 6.25$ m
Wavelength	$\lambda = 525$ nm
Nyquist pixel spacing of phase screens	$\delta = 5.0630$ mm
Nyquist pixel spacing of PSFs	$\delta_f = 2.6895\ \mu$m
Nyquist pixel spacing of object plane	$\delta_o = 0.1290$ m
Pixel spacing of detector (CCD)	$\delta_{\text{CCD}} = 5.5\ \mu$m
Propagation distance through free space	$L_{\text{free}} = 250$ km
Propagation distance through turbulence	$L_{\text{turb}} = 50$ km
Propagation step in turbulence	$\Delta z = 5$ km
Number of phase screens	$N = 11$
Extended phase screen samples	$\mathcal{N}_{\text{ext}} = 3072$ pixels
Local phase screen samples	$\mathcal{N} = 1024$ pixels
Refractive index structure parameter	$C_n^2 = 1.0 \times 10^{-16}\ \text{m}^{-2/3}$
Inner scale	$\ell_0 = 0.01$ m
Outer scale	$L_0 = 300$ m
Object model samples	257×257 pixels
Object plane side length	33.18 m
Pixel skip	4 pixels (65×65 PSF arrays)

3.2 Image recovery from light curve and PSF map

Image recovery from a perfect PSF map

The inputs are the nonresolved light curve and the point spread function map (PSF map) observed at 2000 frames. The test object is the Hubble Space Telescope of size 13.2 meters, located at an altitude of 300 km.

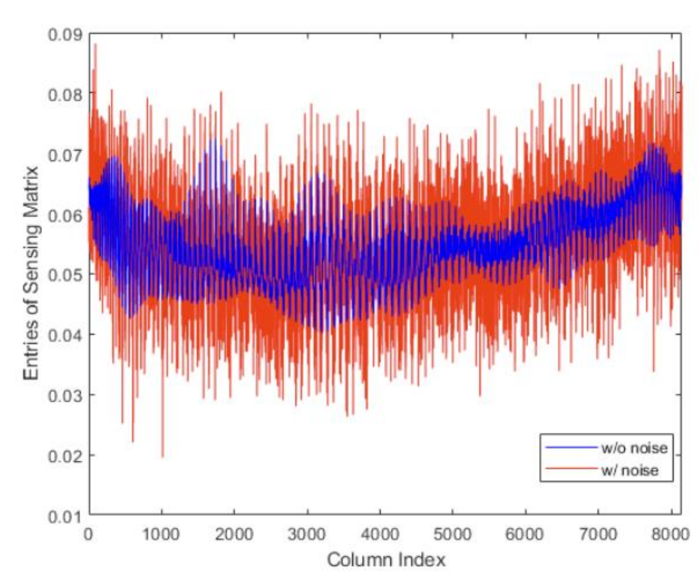


Raw recovered image *Denoised image*

The proposed CS-based method has allowed the recovery of the resolved image with SSIM 0.772. The image quality is enhanced by removing noise by the BM3D algorithm. The denoised image has achieved SSIM 0.932.

Image recovery from a noisy PSF map

In real measurements, exact knowledge of the PSF will not be available. Hence, noise has been introduced to degrade the PSF with levels of 2%, 10%, 15%, 20%, and 25% Gaussian noise. The proposed method still recovers clear images.



Slice of PSF map with 25% noise

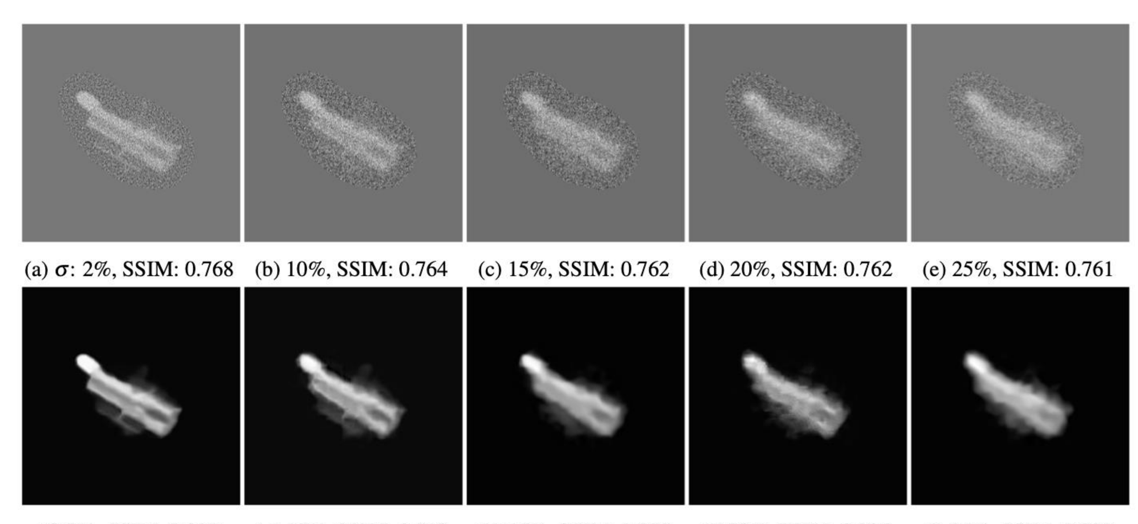


Image recovery from light curve and noisy PSF map. Top: raw recovered images, bottom: denoised images.

4. Conclusions

A new method for characterizing LEO objects has been developed by adapting compressed sensing . The inputs are the object's light curve and the simultaneous measurement of point spread function (PSF) of the atmospheric turbulence. The reconstruction of a fully-resolved object image is possible in test cases, for the 13.2-meter object in 300 km altitude. The proposed approach is successful even when the point spread function is degraded by noise.

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