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On the Analysis and Observability of a Networked Competitive Multi-Virus SIR Model

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Objectives

- Propose an epidemic model which captures the spread of multiple competitive computer viruses over a network
- Provide a sufficient condition which ensures that each computer virus dies out in exponential time
- Estimate the infection level of each computer virus with an observation model

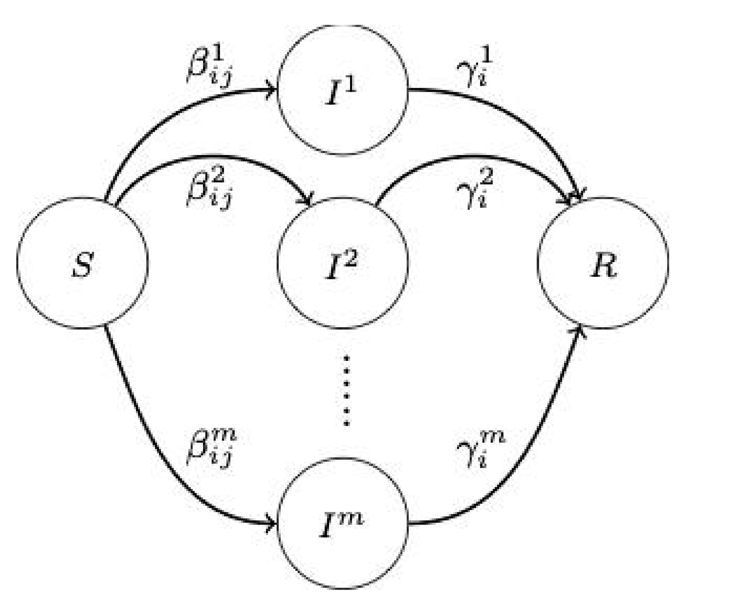
System Model

Networked competitive multi-virus Susceptible-Infected-Recovered (SIR) epidemics model proposed in [1]:

$$s_i[t+1] = s_i[t] - h s_i[t] \sum_{k=1}^{m} \sum_{j=1}^{n} \beta_{ij}^k x_j^k[t]$$

$$x_i^k[t+1] = x_i^k[t] + h\Big(s_i[t] \sum_{j=1}^{n} \beta_{ij}^k x_j^k[t] - \gamma_i^k x_i^k[t]\Big)$$

$$r_i[t+1] = r_i[t] + h \sum_{k=1}^{m} \gamma_i^k x_i^k[t]$$



The competing SIR networked model

Assumption 1

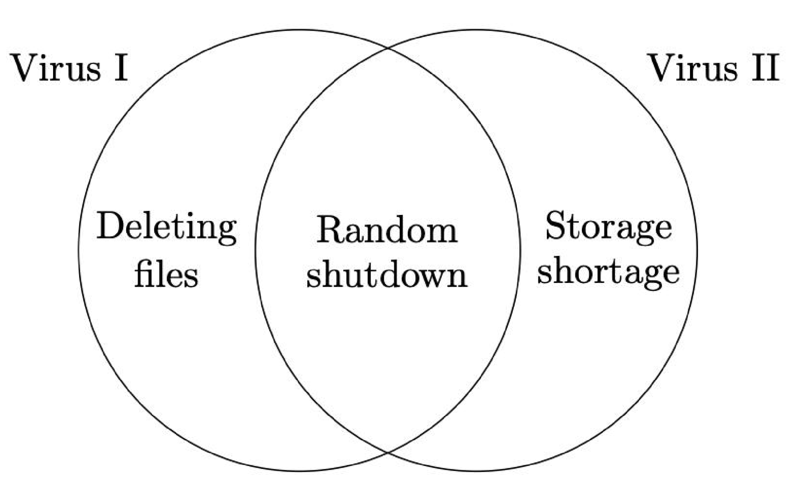
For all $i \in [n]$, $k \in [m]$, we assume that $s_i[0]$, $x_i^k[0]$, $r_i[0] \in [0,1]$, $s_i[0] + \Sigma_{k=1}^m x_i^k[0] + r_i[0] = 1$, $\beta_{ij}^k \geq 0$, $\gamma_i^k > 0$, $h\Sigma_{k=1}^m \Sigma_{j=1}^n \beta_{ij}^k \leq 1$, and $h\Sigma_{k=1}^m \gamma_i^k \leq 1$.

- Let $M^k := I - h\Gamma^k + hB^k$ where, $S[t] = diag(s_i[t])$, B^k is a matrix with (i,j)-th entry β_{ij}^k and $\Gamma^k[t] = diag(\gamma_i^k)$
- We let $\rho(M^k)$ denote the spectral radius of M^k, which can be interpreted as the reproduction number of virus k over the network

Theorem 1

Under Assumption 1, if $\rho(M^k) < 1$, then the k-th virus dies out in exponential time, and this holds for all $k \in [m]$.

State Observation Model



Symptoms of two computer viruses

The observation model is written as

$$y_i[t] = \sum_{k=1}^{m} c_i^k x_i^k[t]$$

where $c_i^k > 0$, for all $i \in [n]$, $k \in [m]$.

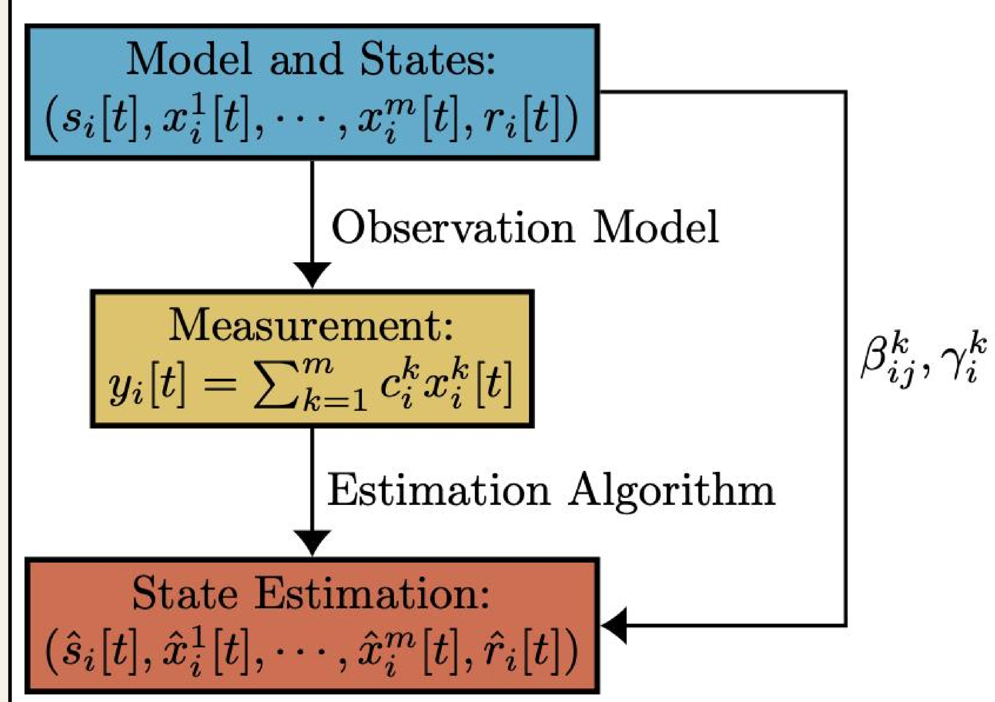
Theorem 2

Under Assumption 1, if, for each $i \in [n]$, γ_i^k is a distinct value for all $k \in [m]$, the competing virus model is locally observable at $s_i[t] = 0$, for all $i \in [n]$.

Estimation Algorithm

We use the following Luenberger observer:

$$\hat{x}_i^k[t+1] = \hat{x}_i^k[t] + h\,(\hat{s}_i[t] \sum_{j=1}^{n} \beta_{ij}^k \hat{x}_j^k[t] - \gamma_i^k \hat{x}_i^k[t]) + L_i(y_i[t] - \hat{y}_i[t])$$



where

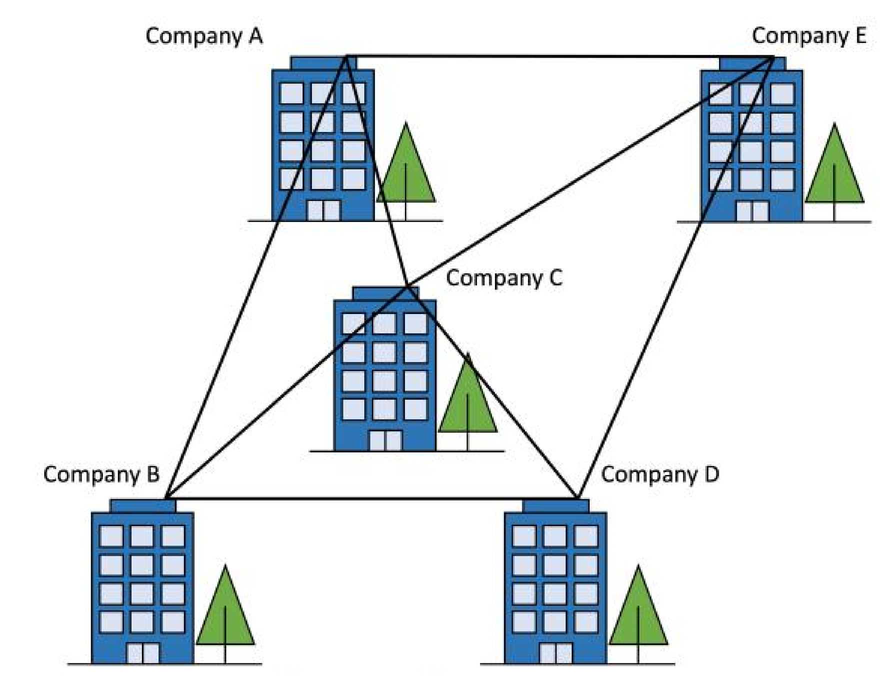
$$\hat{s}_i[t] = 1 - \sum_{k=1}^m \hat{x}_i^k[t] + \hat{r}_i[0]$$

in which the recovered level is estimated through

$$\hat{r}_i[t] = h \sum_q^t \sum_{k=1}^m \gamma_i^k \hat{x}_i^k[q]$$

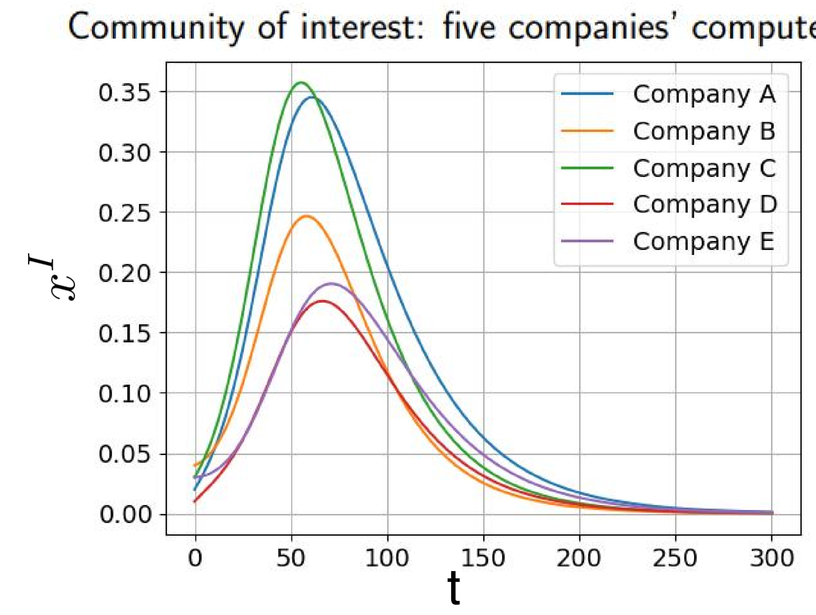
at each time step, recursively.

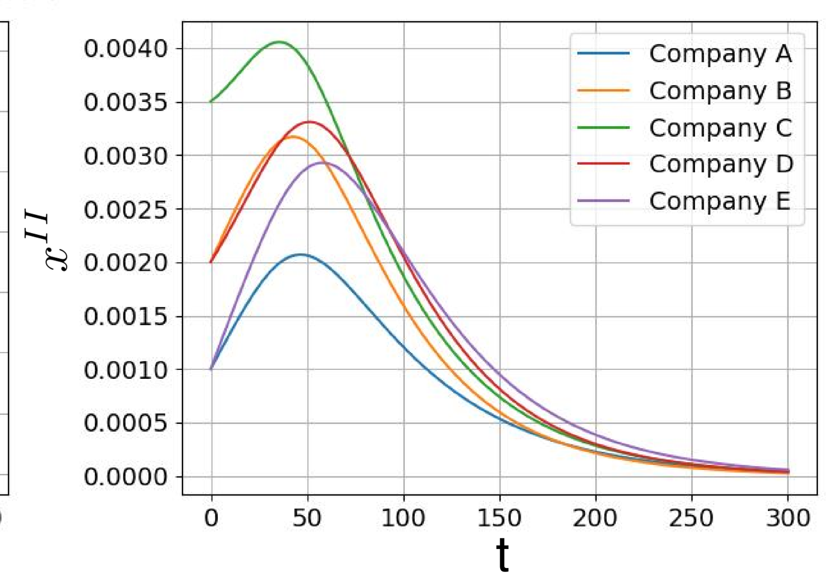
Simulation



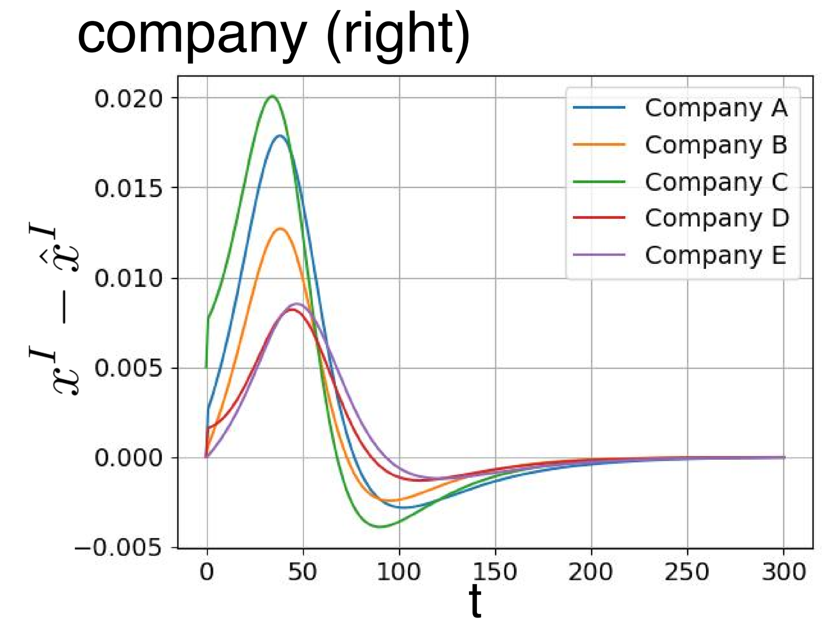
Community of interest: five companies' computers

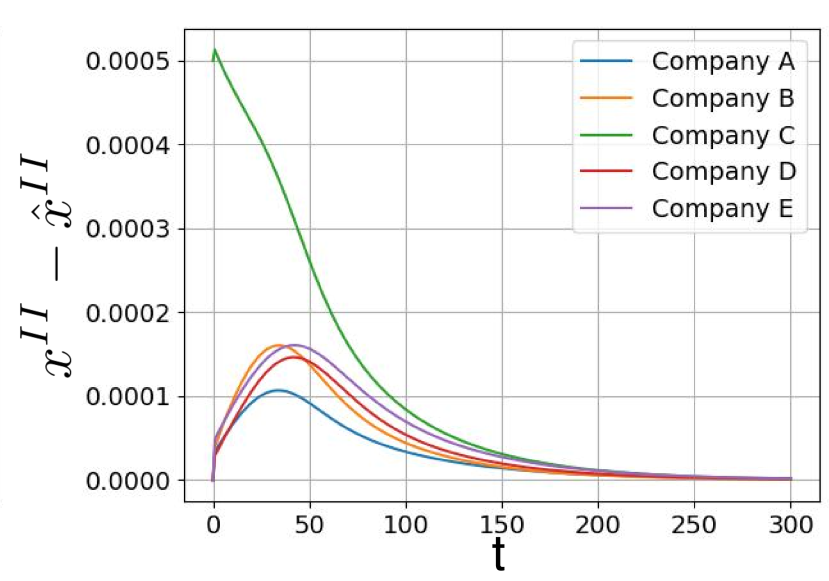
- Both computer viruses die out eventually
- The estimation error converges to zero before the viruses die out
- The magnitude of the estimation error of each virus in each company is less than 10% of its infected proportion





- Evolution of proportion of computers infected by virus I in each company (left)
- Evolution of proportion of computers infected by virus II in each company (right)





- Estimation error of the infected proportion of computers by virus I in each company (left)
- Estimation error of the infected proportion of computers by virus II in each company (right)

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[1] Ciyuan Zhang, Sebin Gracy, Tamer Başar, and Philip E. Paré, "On the Analysis and Observability of a Networked Competitive Multi-Virus SIR Model," submitted to NecSys 2022. This material is based upon work supported in part by the National Science Foundation, grants NSF-ECCS #2032258 and NSF-ECCS #2032321.