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Sustained Space and Cumulative Complexity Trade-offs for Data-Dependent Memory-Hard Functions

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Motivation

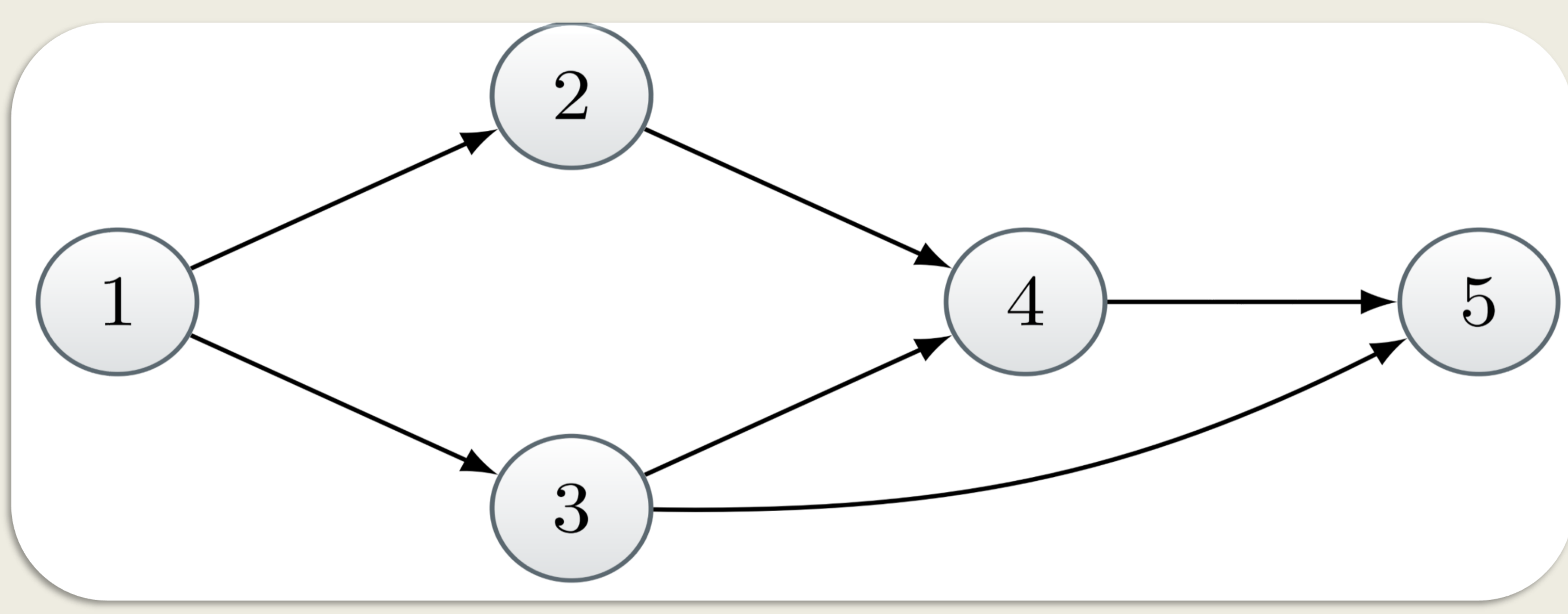
- Billions of user accounts have been affected by password breaches
  - An attacker who obtains hashes of user passwords launches a brute force attack to guess the users' passwords
  - The attacker must evaluate the hash function millions or billions of times
  - Specialized hardware allows attackers to evaluate these functions orders of magnitude faster than standard hardware, but memory cost is relatively uniform across different types of hardware.
- Memory Hard Functions (MHFs) are functions that have high memory cost
- Leaked passwords hashed with MHFs are robust against offline brute-force attacks
- Measures of memory hardness:
  - **Cumulative Complexity (CC):** *The sum (over all steps in the computation) of the memory required to compute the MHF*
  - ***s*-Sustained Space Complexity (*s*-SSC):** *The number of steps required to sustain s bits of memory to computer the MHF.*
- Data-dependent MHFs (dMHFs) are a broad class of MHFs which trade side-channel resistance for easy constructions and asymptotically stronger CC
- **In general, MHFs have weak SSC Guarantees; Can dMHFs perform better?**

Contribution

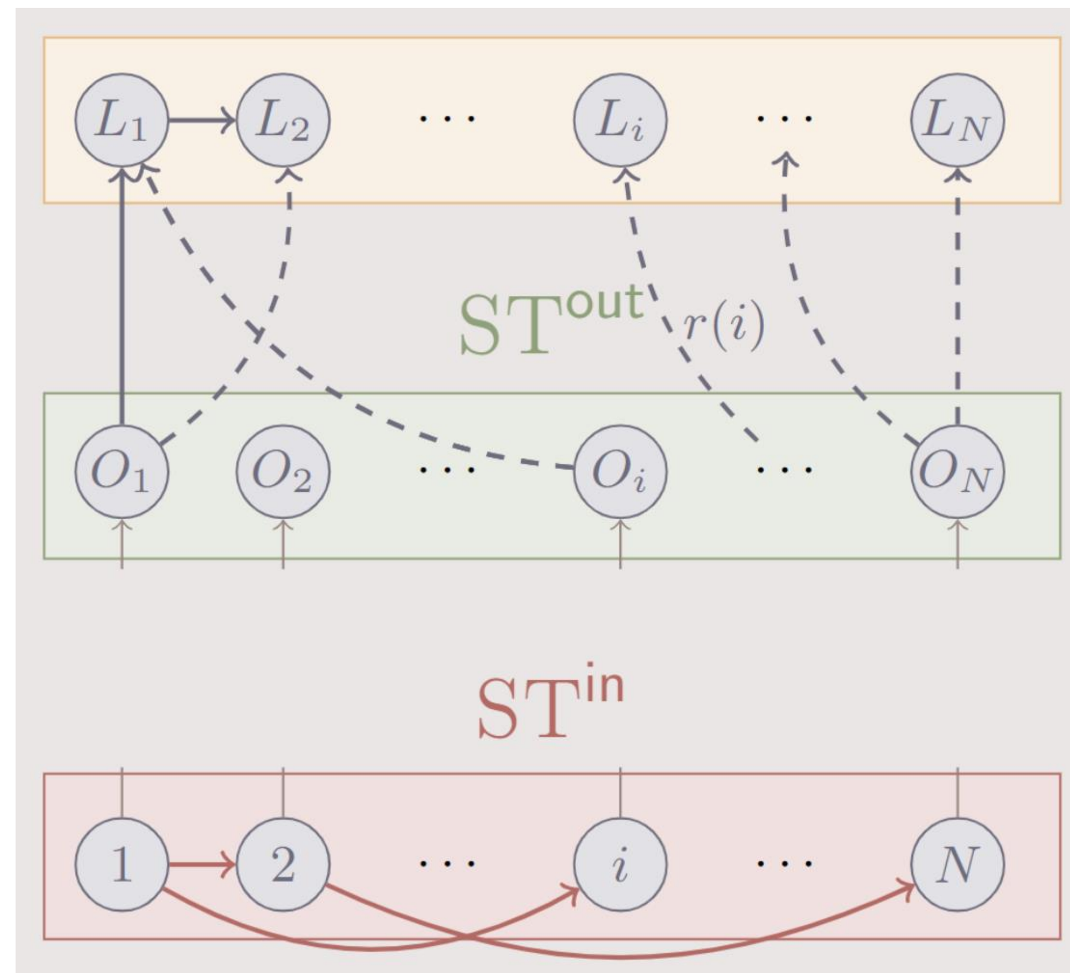
- **dMHFs have much stronger SSC guarantees.**
- We analyze four dMHFs of varying practicality:
  - **Dynamic EGS:** asymptotically maximal SSC, but very impractical
  - **Dynamic DRSample:** practical MHF candidate with almost maximal SSC
  - **Argon2id:** already deployed and widely used, but much weaker SSC than the others
  - **Our Construction**: a theoretical construction with maximal tradeoff with constant indegree

Methods

- Instead analyze pebbling games on graphs
- Each round, pebbles can be placed on any (and all) nodes whose parents are all pebbled
- *Goal: place a pebble on the sink*



- Pebbling: $P_1 = \{1\}, P_2 = \{2,3\}, P_3 = \{3,4\}$, and $P_4 = \{5\}$
- $cc(P) = 1 + 2 + 2 + 1 = 6$
- $2\text{-}ssc(P) = 0 + 1 + 1 + 0 = 2$



Our Construction

- **Dynamic pebbling graphs:** a node v can have a random edge from some prior node $r(v)$ which is only visible to the pebbler once v's predecessor is pebbled
- **ST-Robust graphs:** N inputs/outputs with high connectivity between them
- **Our construction:** $\mathbb{G}_D^N$ (pictured above)
  - ST robust graph with N inputs and N outputs
  - Pebbling graph D overlayed onto the inputs
  - Line graph at the end with random edges to outputs
- **Intuition:** high connectivity between inputs and outputs
  - If a pebbling strategy uses relatively few pebbles, then (with high probability) they need to repebble many inputs
  - The inputs have high CC, so the strategy incurs high cost

Results

- Results of the following form:
  - A pebbling strategy either sustains s pebbles for $\Omega(N)$ steps, or has CC at least C
  - Every graph has CC at most $N^2/2$, so the goal is to require CC $\omega(N^2)$ while keeping s as large as possible
- Graphs with higher than constant indegree lead to MHFs that are Impractical for common applications

Dynamic Graph	Indegree	Sustained Space	Cumulative Cost
Dynamic EGS	$O(\log N)$	$\Omega(N)$	$\Omega(N^3)$
Dynamic DRSample	2	$\Omega\left(N/\log N\right)$	$\Omega\left(N^3/\log N\right)$
Argon2id	2	$\Omega(N^{1-\epsilon})$	$\Omega(N^{2+2\epsilon})$
Our Construction	2	$\Omega(N)$	$\Omega(N^{3-\epsilon})$

- **Open Question:** can we use these pebbling arguments on dynamic graphs directly prove similar SSC/CC trade-offs for their corresponding dMHFs?

References

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