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Resilient Sensor Placement for Kalman Filtering in Diffusion Networks

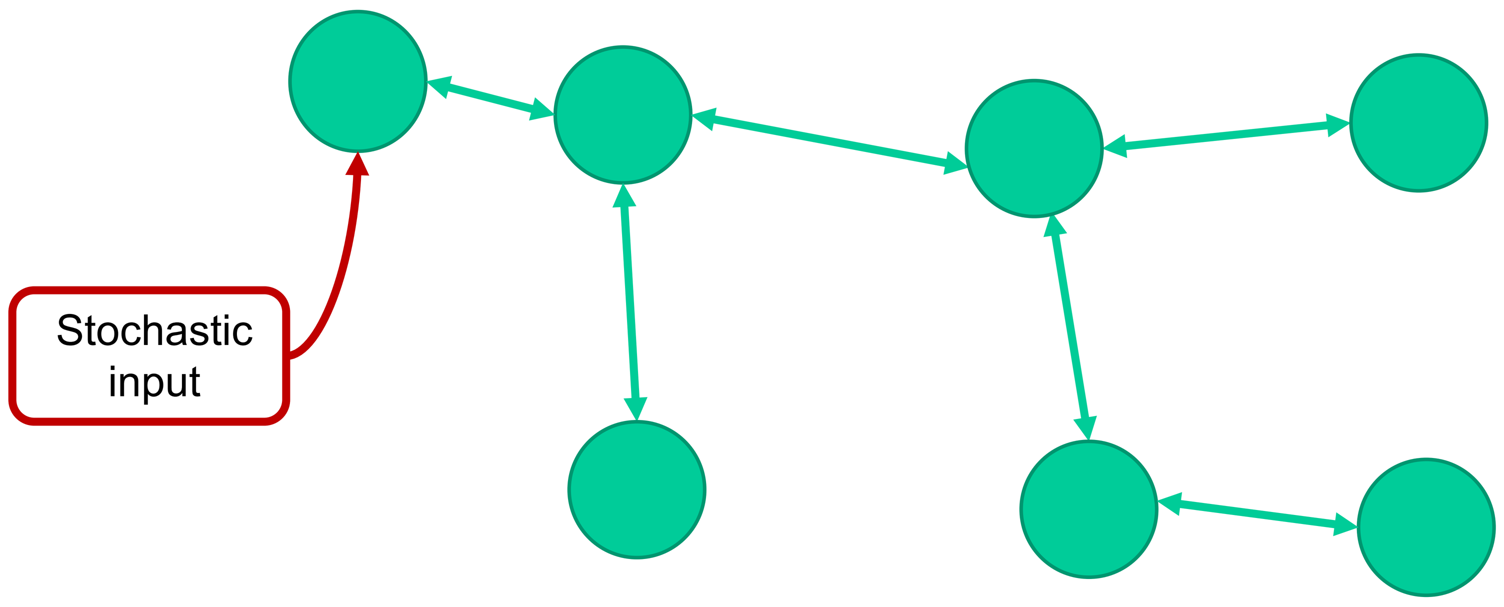
Lintao Ye*, Sandip Roy** and Shreyas Sundaram*

* School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907

**School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164

1. MOTIVATION

- Consider a networked system, where there is single source node affected by an unknown (stochastic) input stream.



- Goal: In order to estimate the states of the above system, sensors can potentially be placed at any nodes in the network.
- Constraint: Due to a budget constraint, only a subset of all the nodes can be chosen to place sensors.
- Key question: How to select an optimal set of the nodes to place sensors, under the budget constraint, with respect to certain performance criterion?
- Tool: We apply the Kalman filter (KF) to estimate the states of the system using the measurements of the sensors, and obtain the mean square estimation error (MSEE) of the KF.

2. THE SENSOR PLACEMENT PROBLEM

- The underlying graph of the networked system is denoted as $G = (V, E)$, where V is the set of nodes and E is the set of edges.
- The dynamics of the source node in the networked system is given by

$$(x_{k+1})_{i_0} = a_{i_0 i_0}(x_k)_{i_0} + \sum_{j \in \mathcal{N}_{i_0}} a_{i_0 j}(x_k)_j + w_k,$$

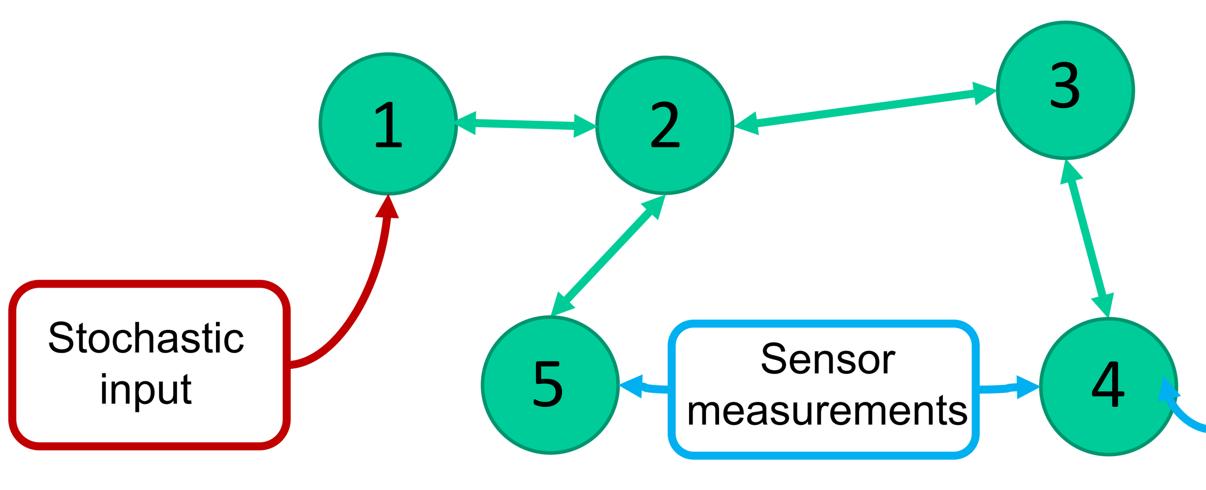
where i_0 is the source node, $a_{i_0 i_0}$ and $a_{i_0 j}$ are the specified by the dynamics, $\mathcal{N}_{i_0}$ is the set of neighbors of node i_0, and w_k is the input stream. Similarly, the dynamics of the other nodes are given by

$$(x_{k+1})_i = a_{ii}(x_k)_i + \sum_{j \in \mathcal{N}_i} a_{ij}(x_k)_j.$$

- A sensor placed at node j gives a measurement of the state of node j:

$$(y_k)_j = (x_k)_j.$$

- Consider a *0-1* indicator vector μ that indicates which nodes are chosen to place sensors. An illustrative example: $\mu = [0\ 0\ 0\ 1\ 1]^T$, when there are five nodes (labeled as follows) in the networked system.



- Placement costs of the sensors are gathered in a vector: $h = [h_1, \ldots, h_n]^T$.

- Problem 1: Consider a graph $G = (V, E)$ for the network, a source node i_0, the sensor placement cost of each node and the sensor placement budget H. The Graph-based Kalman Filtering Sensor Placement (GKFSP) problem is to solve

$$\min_{\mu} trace(\Sigma(\mu))$$
$$s.t.\ u^T h \leq H.$$

MSEE of the KF

- By leveraging the graph structure of the networked system, we obtain:
- Main result: The optimal solution to GKFSP can be achieved by putting a *single* sensor at the node that is as *close* as possible to the source node while satisfying the placement budget constraint.
- Algorithm: Such a node can be found via a shortest-path algorithm for graphs running in polynomial time.

Reference: Part of the work about the GKFSP problem was presented in:
L. Ye, S. Roy and S. Sundaram, "Optimal Sensor Placement for Kalman Filtering in Stochastically Forced Consensus Networks" *in Proc.* IEEE Conference on Decision and Control, FL, 2018, 6686-6691.

3. THE SENSOR ATTACK PROBLEM

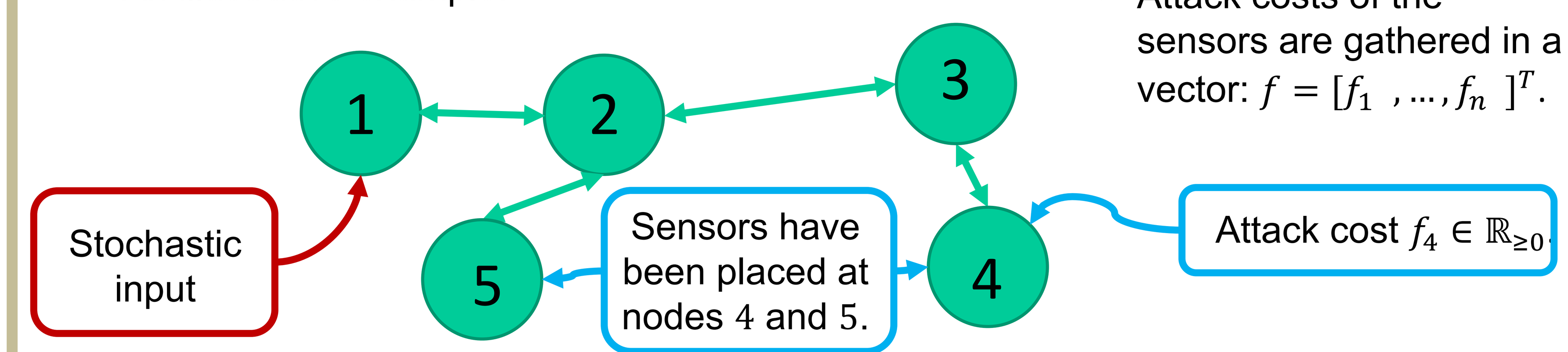
- Consider the scenario where the system designer has already placed sensor(s) among the node(s) in the network. An adversary aims to remove the placed sensor(s) to corrupt the estimation performance of the system states under an attack budget constraint.
- Problem 2: Consider a graph $G = (V, E)$ for the network, a source node i_0, a sensor placement vector μ, the sensor attack cost of each node and the sensor attack budget constraint Ω. The Graph-based Kalman Filtering Sensor Attack (GKFSA) problem is to solve

$$\max_{v} trace(\Sigma(\mu \backslash v))$$
$$s.t.\ v^T f \leq \Omega,$$

- $\mu \backslash v$ indicates the surviving sensors after the attack.

where f is the attack cost vector and v is the sensor attack vector that indicates the targeted sensors.

- An illustrative example:



- Attack costs of the sensors are gathered in a vector: $f = [f_1, \ldots, f_n]^T$.

A sensor attack vector $v = [0\ 0\ 0\ 1\ 0]^T$ indicates targeting the sensor placed at node 4.

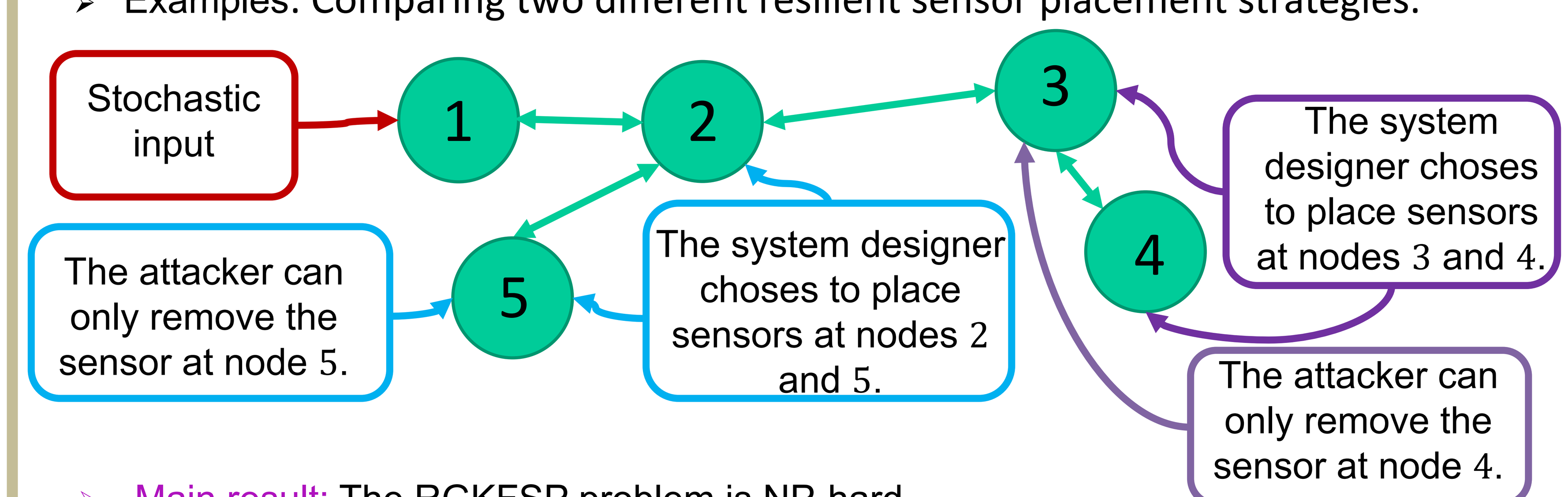
- Main result: Among the set of placed sensors, an optimal solution to GKFSA can be achieved by targeting the sensors that are closer to the source node while satisfying the attack budget constraint.
- Algorithm: Such a sensor attack can be found via a shortest-path algorithm for graphs running in polynomial time.

4. THE RESILIENT SENSOR PLACEMENT PROBLEM

- Next, consider the scenario where the system designer is aware of the potential attacks from the adversary and seeks to find an optimal sensor placement that yields the minimum MSEE of the KF after the adversary's actions.
- Problem 3: Consider a graph $G = (V, E)$ for the network, a source node i_0, the sensor placement cost vector h, the sensor attack cost vector f, the sensor placement budget H and the sensor attack budget Ω. The Resilient Graph-based Kalman Filtering Sensor Placement (RGKFSP) problem is to solve

$$\min_{\mu} \max_{v} trace(\Sigma(\mu \backslash v))$$
$$s.t. u^T h \leq H,$$
$$v^T f \leq \Omega.$$

- Examples: Comparing two different resilient sensor placement strategies.



- Main result: The RGKFSP problem is NP-hard.
- Explanation: There is no polynomial-time algorithm for RGKFSP unless P=NP.
- Algorithm: An optimal solution to RGKFSP can be found based on a dynamical-programming approach.
- We achieve this by relating the RGKFSP problem to knapsack problems.

5. OUR CONTRIBUTRIONS

- We provide polynomial-time algorithms for the GKFSP and GKFSA problems via leveraging the structure of the underlying graph of the networked system.
- We prove that the RGKFSP problem is NP-hard.
- We give an algorithm based on dynamical programming to solve the RGKFSP problem.

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