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The Center for Education and Research in Information Assurance and Security

Secure Distributed State Estimation for Large-Scale Systems

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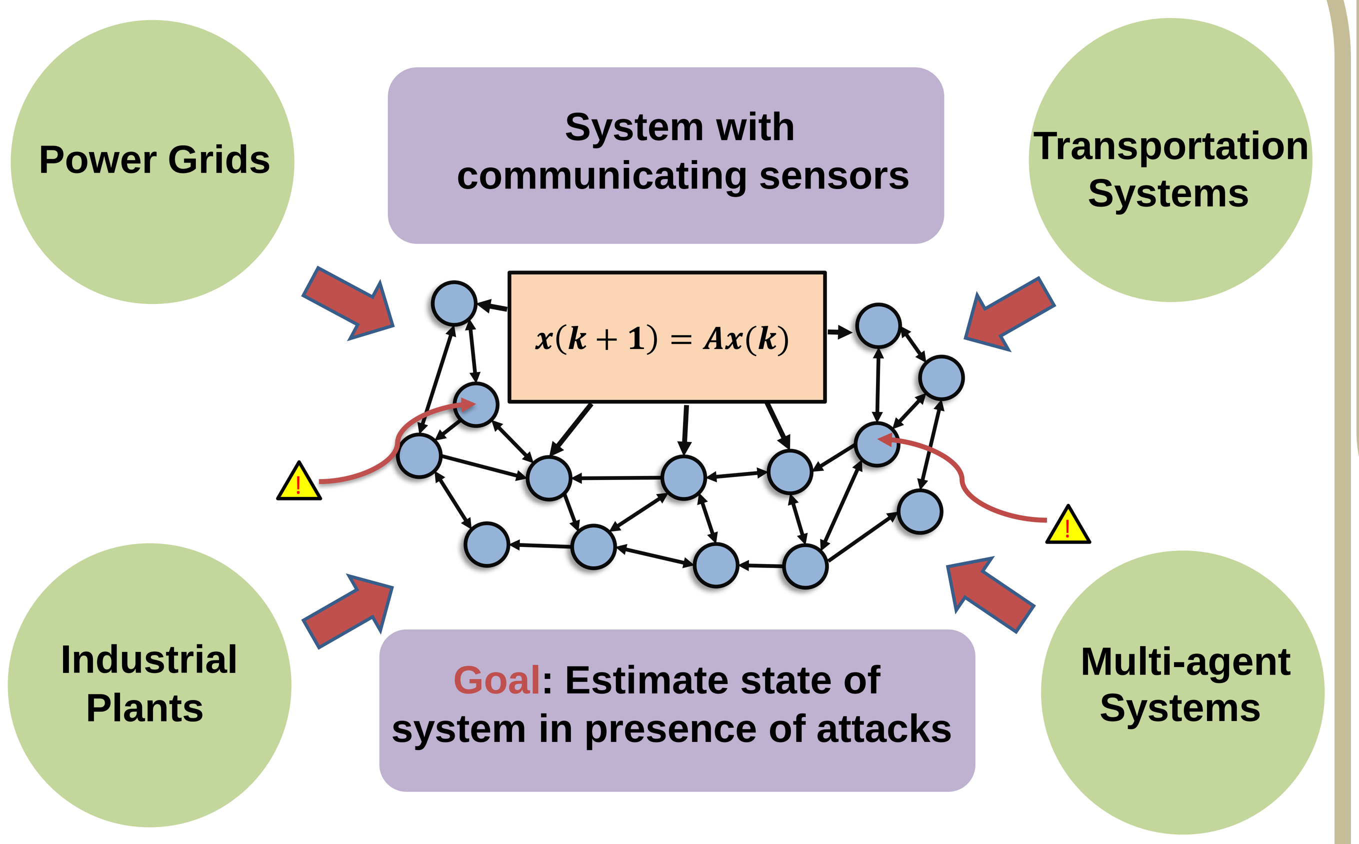
1. MOTIVATION



- To control complex systems (e.g. power grids, gas turbines, transportation systems), we need to monitor their state dynamics.
- Consider an LTI dynamical system whose state $x(k)$ is monitored by a network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of N sensor nodes.
- Each node i has measurement $y_i(k) = C_i x(k)$.
- Objective: Each node must estimate the state $x(k)$ (asymptotically).
- Key Question: What if some sensors are compromised by adversaries?
  - Can have catastrophic effects in safety critical systems.
  - Must develop secure distributed state estimation algorithms.

2. ADVERSARY MODEL

- We consider worst-case adversarial behavior (Byzantine adversaries).
- Adversaries have complete knowledge of system dynamics and communication network.
- Adversaries can deviate arbitrarily from any prescribed algorithm.
- In contrast, regular (non-adversarial) nodes possess only local information.
- At most f adversaries in the neighborhood of any regular node (f-local model).

3. OUR CONTRIBUTION

- We develop a secure distributed state estimation algorithm with provable guarantees.
- Algorithm is lightweight, fully distributed and runs on a single-time-scale.
- We characterize feasible network topologies that guarantee success of our algorithm.

4. ESTIMATION STRATEGY

- Each node estimates a portion of the state using local measurements.
- For estimating the rest of the state, it communicates with neighbors.
- Key is to identify "leaders" or "source nodes" for certain portions of the state.
- Disseminate information from source nodes to the rest of the network via "sufficient number of redundant" paths.

5. LOCAL LUENBERGER OBSERVERS

- For identifying source nodes, diagonalize the system: $z(k) = V^{-1}x(k)$.
- Each node i constructs a local Luenberger observer

$$\hat{z}_{o_i}(k+1) = J_i\, \hat{z}_{o_i}(k) + L_i\left(y_i(k) - C_{o_i}\hat{z}_{o_i}(k)\right)$$

Contains eigenvalues node i can detect; Portion of state node i can estimate independently; Locally designed gain matrix

Local observer guarantees that $\hat{z}_{o_i}(k+1)$ converges to true value.

- The tricky part: Node i has to communicate with neighbors (can be adversarial) to estimate the rest of the state. Dependent on certain "source nodes" for this purpose.

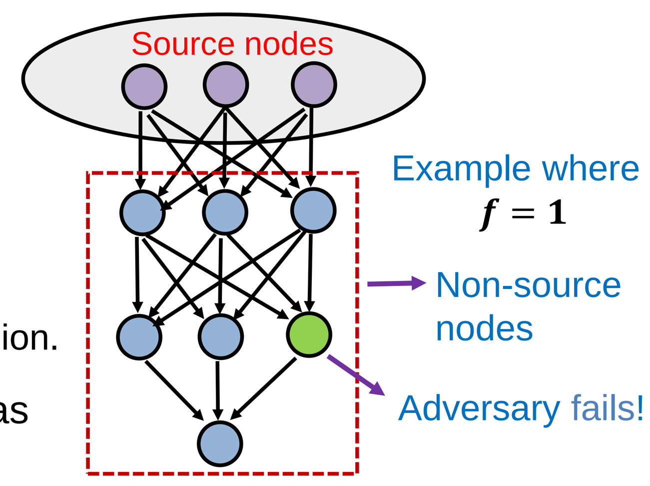
6. SECURE CONSENSUS SCHEME

Question: How to transmit info from source nodes to the rest of the network securely?

Solution: Construct Mode Estimation Directed Acyclic Graphs (MEDAGs)

MEDAGs provide sufficient number of redundant paths for information transmission.

In particular, each non-source node has $(2f+1)$ parents in a MEDAG.



MEDAG construction algorithms are fully distributed and resilient to adversarial attacks.

7. LOCAL FILTERING DYNAMICS

- Suppose node i cannot estimate the state $z_j(k)$.
- Let S_j denote the set of source nodes that can estimate $z_j(k)$.
- Construct MEDAG to transmit info from S_j to node i.
- At each time-step k, node i collects estimates from its parents in the MEDAG.
- It throws away the highest and lowest f estimates, and takes a convex combination of the rest of the estimates as follows:

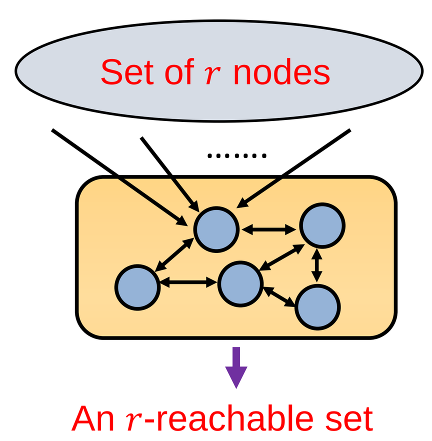
$$\hat{z}_i^{(j)}(k+1) = \lambda_j \sum_{v_l \in \mathcal{M}_i(k)} w_{il}^{j}\hat{z}_l^{(j)}(k)$$

Unstable eigenvalue that node i cannot detect; Convex weights; Parents after removing extreme values

KEY IDEA: Ignore extreme values in neighborhood, take weighted average of the rest.

8. FEASIBLE NETWORK TOPOLOGIES

- Feasible networks allow success of our algorithm.
- They are characterized by a graph property known as "robustness".
- Robustness is defined in terms of r-reachable sets.
- A set S of nodes is r- reachable if it contains at least one node that has r neighbors outside the set.



MAIN RESULT: If the network is "strongly (3f+1)-robust" w.r.t. every set of source nodes, then all regular nodes can asymptotically estimate the state using the proposed secure distributed state estimation technique, despite the actions of any f- local set of adversaries.

9. OVERALL TECHNIQUE

DESIGN PHASE

- Identify source nodes.
- Construct MEDAGs to transmit info from source nodes to rest of the network.

ESTIMATION PHASE

- Each node uses local Luenberger observer to estimate portion of state.
- For the rest of the state, it listens to its parents in the MEDAG and applies the local filtering scheme.

10. TAKE-AWAY POINTS

- Real-time control of complex systems requires precise estimation of state dynamics via sensor measurements.
- Investigated the problem where certain sensors are compromised by adversaries.
- Developed a lightweight, fully distributed secure state estimation algorithm with provable guarantees against worst-case attacks.
- Characterized communication networks that facilitate our method.

11. REFERENCE

A. Mitra and S. Sundaram, "*Secure Distributed Observers for a Class of Linear Time Invariant Systems in the Presence of Byzantine Adversaries,*" IEEE CDC 2016

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