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Distributed Fault Detection and Isolation for Kalman Consensus Filter

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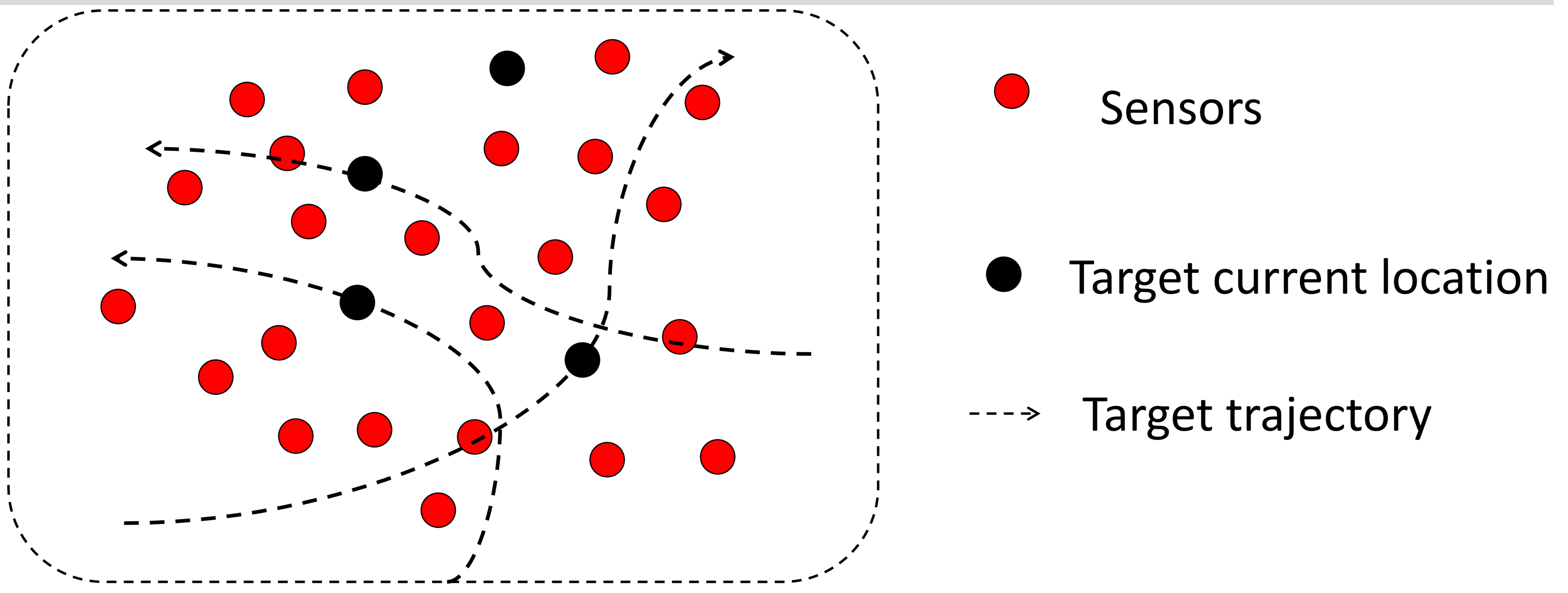
Overview

OBJECTIVE – Track targets in a distributed manner via a sensor network in the presence of faults (cyber-attacks)



Faults/Cyber attacks in Sensor Network

Cyber attacks can cause an unacceptable performance in the surveillance parameter of the sensor.

Surveillance Parameter	Fault	Stochastic Language
Latency	Delayed transmission/reception	Bias
Accuracy	Transmission of states with noise	Covariance
Integrity	Incorrect Data	Spike
Continuity	No transmission/ reception	No data

Current Algorithms for Target Tracking

Target Dynamic Equation $x(k+1) = A(k)x(k) + B(k)\gamma(k);$

$x(0) \sim \mathcal{N}(0, P_0), \gamma(k) \sim \mathcal{N}(0, Q)$

Sensor Model $z(k) = H(k)x(k) + v(k) \qquad v(k) \sim \mathcal{N}(0, R)$

CENTRALIZED KALMAN FILTER

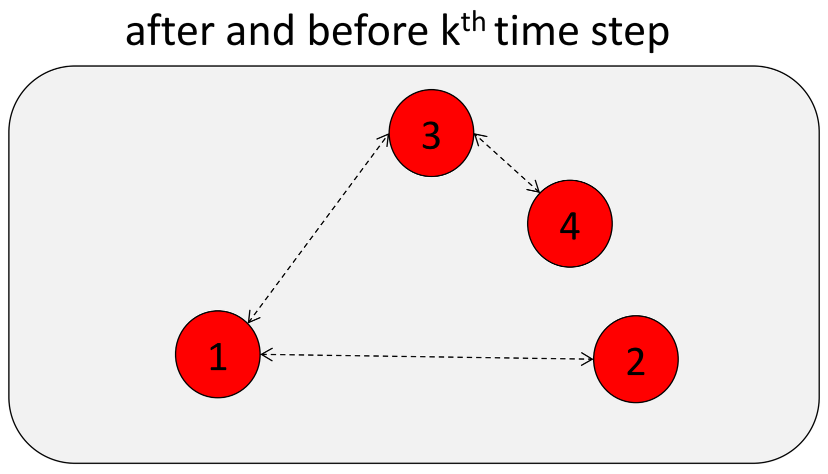
Update Phase

$$\begin{cases} \hat{x}^+(k) = \hat{x}^-(k) + M\left(y - S\hat{x}^-(k)\right) \\ S = (H_1)^T(R_1)^{-1}H_1 + (H_2)^T(R_2)^{-1}H_2 \\ y = (H_1)^T(R_1)^{-1}z_1 + (H_2)^T(R_2)^{-1}z_2 \\ M = (W+S)^{-1} \end{cases}$$

Predict Phase

$$\begin{cases} W(k+1) = (AMA^T + BQB^T)^{-1} \\ \hat{x}^-(k+1) \leftarrow A\hat{x}_i^+(k) \end{cases}$$

Notation: (x^+) and (x^-) : It denote estimate of x after and before k^{th} time step



Sensor Network

KALMAN CONSENSUS FILTER

Step 1: Each node will calculate u_i and U_i and transmit it to the neighbor.

$$u_i = (H_i)^T(R_i)^{-1}z_i$$
$$U_i = (H_i)^T(R_i)^{-1}H_i$$

Step 2: Fuse information: .

$$y_i = \sum_{i' \in \mathcal{N}_{C_i} \cup \{i\}} u_{i'}$$
$$S_i = \sum_{i' \in \mathcal{N}_{C_i} \cup \{i\}} U_{i'}$$

Step 3: Update the estimate and add a consensus term

$$\hat{x}_i^+(k) = \hat{x}_i^-(k) + M_i\left(y_i - S_i\hat{x}_i^-(k)\right) \quad \leftarrow \text{Standard Kalman Update}$$
$$+ \gamma W_i(k)^{-1} \sum_{i' \in \mathcal{N}_{C_i}} \left(\hat{x}_{i'}^-(k) - \hat{x}_i^-(k)\right) \quad \leftarrow \text{Consensus term}$$
$$M_i = (W_i + S_i)^{-1}$$

Step 4: Predict the next position (same as KF)

Fault detection Techniques

Method 1: Covariance Matching

Step 1: Calculate residue and **theoretical covariance** of the residue.

$$r(k) = z(k) - H\hat{x}^-(k)$$
$$C_r(k) = H(W^-(k))^{-1}H^T + R$$

Step 2: Calculate **sample covariance** of residue by some previous measurements (say k)

$$\hat{C}_r(k) = \frac{1}{L}\sum_{i=k-L+1}^{k} r(i)r(i)^T$$

Step 3: Define a parameter called **residual compatibility**

$$rC^p(k) = \frac{|r^p(k)|}{\sqrt{C_r(k)(p,p)}}$$

Step 4: Check the conditions on rC **(sensor is good if rC ~ 1)**

Note: Measurement Data of only one sensor is required for this method

Method 2: Consistency Checking

Step 1: Calculate difference parameter

$$D_{12} = |z_1 - z_2|), D_{13} = |z_1 - z_2|), D_{14} = |z_1 - z_4|)$$
$$D_{23} = |z_2 - z_3|), D_{24} = |z_2 - z_4|), D_{34} = |z_3 - z_4|)$$

Step 2: if D_{12}, D_{13} and D_{14} are greater than a threshold then sensor 1 is faulty. Similar checks can be introduced for other sensors too.

Note: Measurement Data of more than one sensor is required for this method

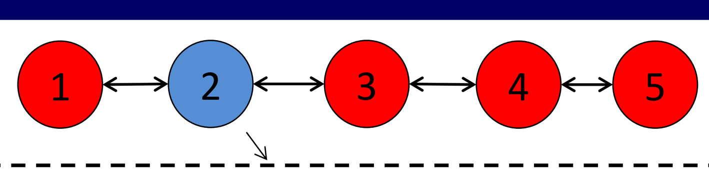
ROBUST KALMAN CONSENSUS FILTER

Spike Detection: Use method 1 and method 2 on each node.

Covariance Detection: Introduce a separate Kalman Filter on each node and use method 2

No Data: KCF works with asynchronous communication

Results

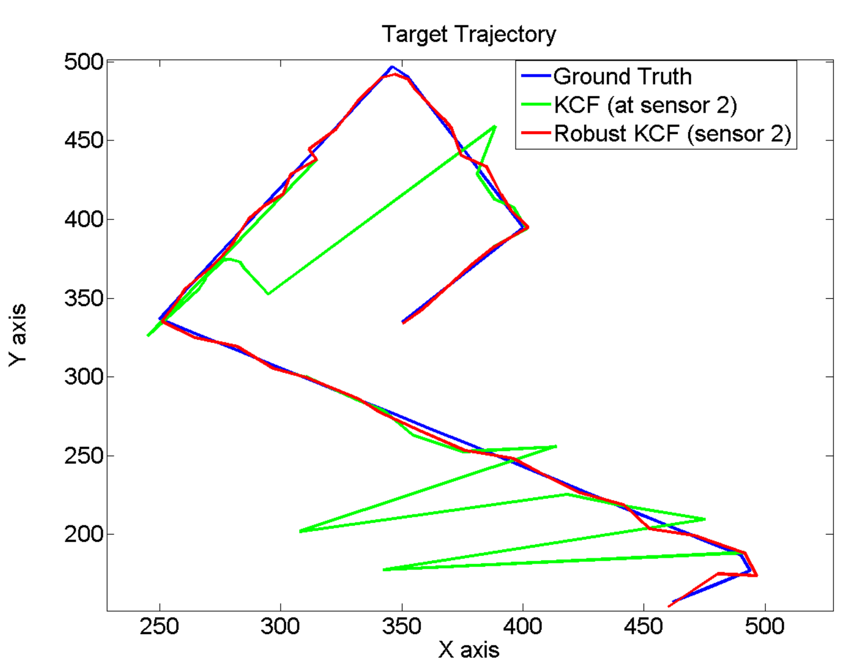


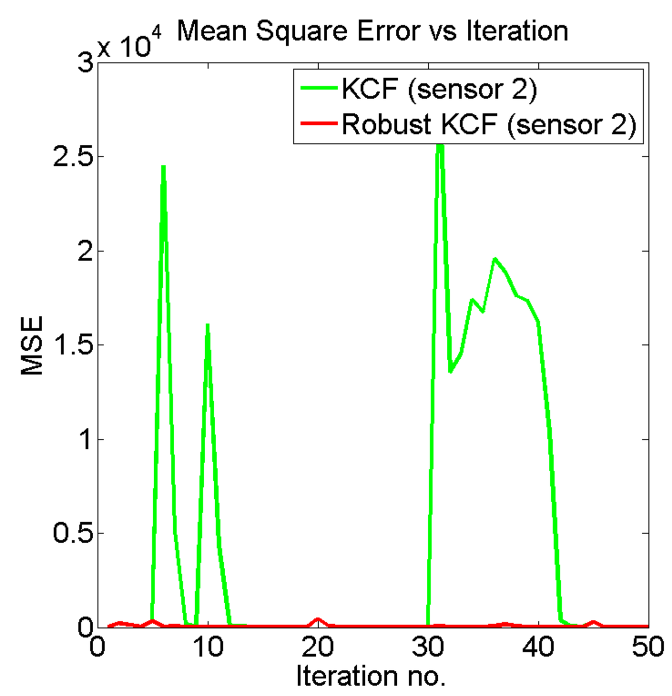
Faulty Sensor: Constant Fault (a constant value) at iteration no. 6, 10 , 31-40

- Field of View of all the sensors: 500 x 500 grid
- Target is tracked for 50 time instants

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix},$$

$$A = \begin{bmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 1 & 0 & 0.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \mathbf{I_4}, Q = \begin{bmatrix} 30 & 3 & 3 & 3 \\ 3 & 30 & 3 & 3 \\ 3 & 3 & 30 & 3 \\ 3 & 3 & 3 & 30 \end{bmatrix}$$





Contact and references

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