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Explicit formulas for real hyperelliptic curves of genus 2 in affine representation

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Motivation

• To extend elliptic curve based cryptographic protocols and cryptosystems

• To find faster arithmetic to compete with elliptic curves and imaginary hyperelliptic curves while obtaining the same security level

Explicit Formulas Developed

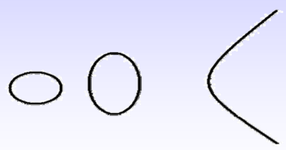
• Baby step

• Divisor addition

• Divisor doubling

Real hyperelliptic curve of genus 2 over finite fields *GF(q)*

C: $y^2+h(x)y=f(x)$



• $y^2+h(x)y-f(x)$ absolutely irreducible, non-singular

• If *q* odd:$f(x)$ monic, deg$(f)=6$, $h(x)=0$

• If *q* even: $h(x)$ monic, $deg(h)=3$, either $deg(f)<6$ or $deg(f)=6$ and $f(x)$ has leading coefficient of form e^2+e for some $e \in GF(q)^*$

Comparison of operation counts for explicit formulas

Notation for operations in finite fields:
I: inversion, S: squaring, M: multiplication

	Imaginary	**Real**
Baby Step	NA	1I, 2S, 4M
Addition	1I, 2S, 22M	1I, 2S, 26M
Doubling	1I, 5S, 22M	1I, 4S, 28M

Diffie-Hellman Key Exchange with real hyperelliptic curves

• Key space: subset of reduced principal ideals in the ring of regular function of *C* with infrastructure; one-to-one correspondence to a subset of divisor class groups of *C*

• Mumford representation and Cantor's algorithm

• Main steps of divisor arithmetic:

- giant step: divisor/ideal composition and reduction

- baby step: output adjustment

Experimental result

Scalar multiplication and key exchange timings over GF(q) (in seconds)

Security Level (Bits)	**Imag**	**Fixed**	**Var**	**DH Imag**	**DH Real**
80	0.0048	0.0050	0.0056	0.0097	0.0106
112	0.0083	0.0085	0.0096	0.0166	0.0180
128	0.0103	0.0106	0.0117	0.0206	0.0223
192	0.0220	0.0230	0.0256	0.0442	0.0485
256	0.0403	0.0411	0.0452	0.0806	0.0863

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